Neutrinos in the Electroweak Theory

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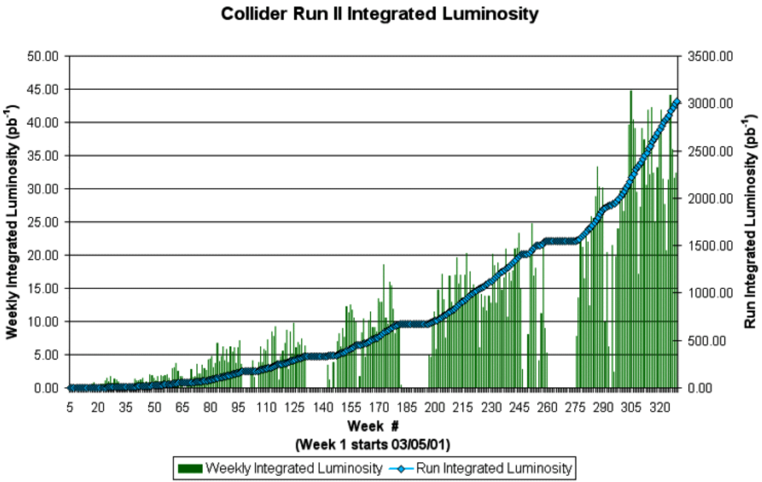
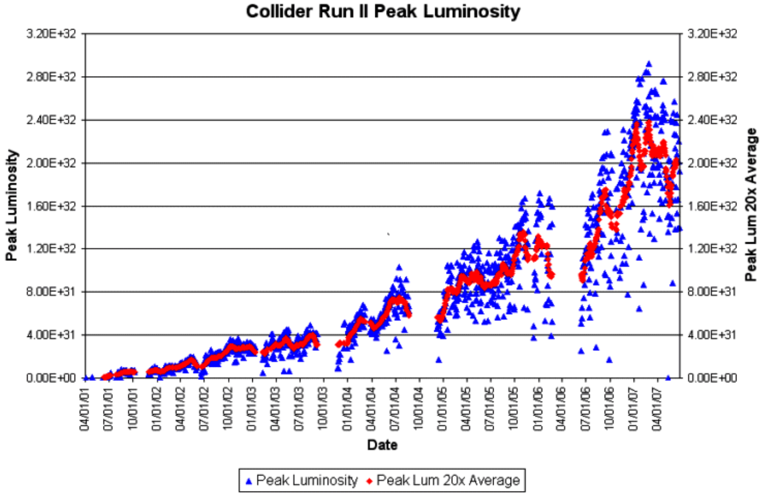
A Decade of Discovery Past . . .

- EW theory → law of nature [Z, e^+e^-, $\bar{p}p$, νN, $(g-2)_\mu, \ldots$]
- Higgs-boson influence in the vacuum [EW experiments]
- ν oscillations: $\nu_\mu \to \nu_\tau$, $\nu_e \to \nu_\mu/\nu_\tau$ [$\nu_\odot$, ν_{atm}, reactors]
- Understanding QCD [heavy flavor, Z^0, $\bar{p}p$, νN, ep, ions, lattice]
- Discovery of top quark [$\bar{p}p$]
- Direct $\mathcal{CP}$ violation in $K \to \pi\pi$ [fixed-target]
- B-meson decays violate $\mathcal{CP}$ [$e^+e^- \to B\bar{B}$]
- Flat universe: dark matter, energy [SN Ia, CMB, LSS]
- Detection of ν_τ interactions [fixed-target]
- Quarks, leptons structureless at 1 TeV scale [mostly colliders]

Tevatron Collider is breaking new ground in sensitivity


Collider Run II Integrated Luminosity
Weekly Integrated Luminosity (pb-1)
Run Integrated Luminosity (pb-1)
50.00
45.00
40.00
35.00
30.00
25.00
20.00
15.00
10.00
5.00
0.00
3500.00
3000.00
2500.00
2000.00
1500.00
1000.00
500.00
0.00
5 20 35 50 65 80 95 110 125 140 155 170 185 200 215 230 245 260 275 290 305 320
Week #
(Week 1 starts 03/05/01)
Weekly Integrated Luminosity
Run Integrated Luminosity



Collider Run II Peak Luminosity
3.20E+32
2.80E+32
2.40E+32
2.00E+32
1.60E+32
1.20E+32
8.00E+31
4.00E+31
0.00E+00
Peak Luminosity
Peak Lum 20x Average
04/01/01
07/01/01
10/01/01
01/01/02
04/01/02
07/01/02
10/01/02
01/01/03
04/01/03
07/01/03
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07/01/05
10/01/05
01/01/06
04/01/06
07/01/06
10/01/06
01/01/07
04/01/07
Date
Peak Luminosity
Peak Lum 20x Average

Tevatron Collider in a Nutshell

980-GeV protons, antiprotons (2π km)

frequency of revolution $\approx 45\,000\ \mathrm{s}^{-1}$

392 ns between crossings

($36 \otimes 36$ bunches)

collision rate $= \mathcal{L} \cdot \sigma_{\text{inelastic}} \approx 10^7\ \mathrm{s}^{-1}$

$c \approx 10^9$ km/h; $\quad v_p \approx c - 495$ km/h

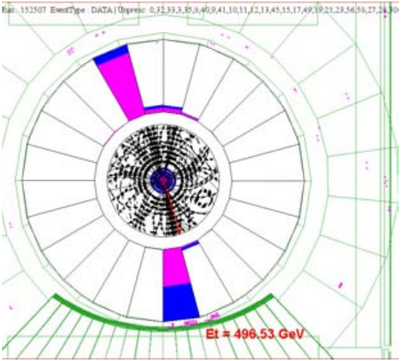
Record $\mathcal{L}_{\text{init}} = 2.85 \times 10^{32}\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1}$

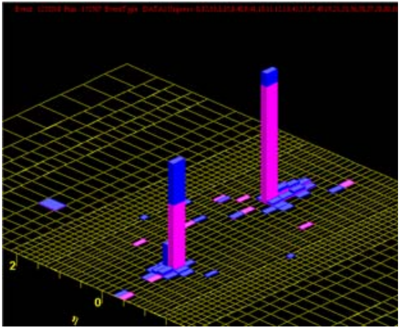
[CERN ISR: pp, $1.4 \times 10^{32}\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1}$]

Goal: $\approx 8\ \mathrm{fb}^{-1}$ by 10.2009

The World's Most Powerful Microscopes

nanonanophysics

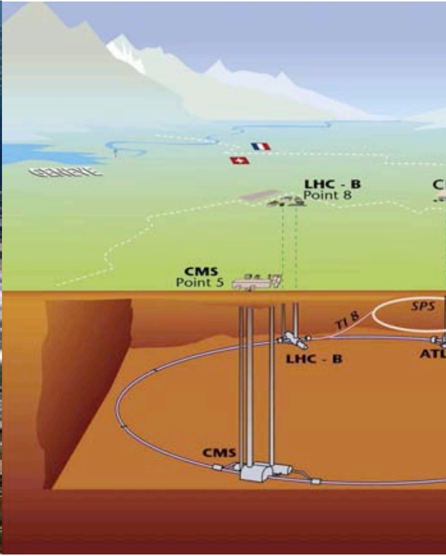




CDF dijet event ($\sqrt{s} = 1.96$ TeV): $E_T = 1.364$ TeV

$$\boxed{q\bar{q} \longrightarrow \text{jet} + \text{jet}}$$

LHC will operate *soon*, breaking new ground in E & $\mathcal{L}$



LHC in a nutshell

7-TeV protons on protons (27 km); $v_p \approx c - 10$ km/h

Novel two-in-one dipoles (≈ 9 teslas)

First collisions at $E_{cm} = 14$ TeV: May 2008

First physics run! Goal of $\gtrsim 1\ \mathrm{fb}^{-1}$ by end 2008

Eventual: $\mathcal{L} \gtrsim 10^{34}\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1}$: $100\ \mathrm{fb}^{-1}/\mathrm{year}$

Why the LHC is so exciting (I)

- Even low luminosity opens vast new realm:
  10 pb^{-1} *(few days at initial $\mathcal{L}$)* yields
  8000 top quarks, 10^5 W-bosons,
  100 QCD dijets beyond Tevatron kinematic limit
  Supersymmetry hints recorded in a few weeks ?
- Essential first step: rediscover the standard model
- The antithesis of a one-experiment machine;
  enormous scope and versatility beyond high-$p_\perp$
- $\mathcal{L}$ upgrade extends $\gtrsim$10-year program . . .

The importance of the 1-TeV scale

EW theory does not predict Higgs-boson mass

▷ Conditional *upper bound* from Unitarity

Compute amplitudes $\mathcal{M}$ for gauge boson scattering at high energies, make a partial-wave decomposition

$$\mathcal{M}(s,t) = 16\pi \sum_J (2J+1) a_J(s) P_J(\cos\theta)$$

Most channels decouple – pw amplitudes are small at all energies (except very near the particle poles, or at exponentially large energies) – $\forall M_H$.

Four interesting channels:

$$W_L^+ W_L^- \quad Z_L^0 Z_L^0/\sqrt{2} \quad HH/\sqrt{2} \quad HZ_L^0$$

L: longitudinal, $1/\sqrt{2}$ for identical particles

In HE limit,[1] s-wave amplitudes $\propto G_F M_H^2$

$$\lim_{s \gg M_H^2} (a_0) \to \frac{-G_F M_H^2}{4\pi\sqrt{2}} \cdot \begin{bmatrix} 1 & 1/\sqrt{8} & 1/\sqrt{8} & 0 \\ 1/\sqrt{8} & 3/4 & 1/4 & 0 \\ 1/\sqrt{8} & 1/4 & 3/4 & 0 \\ 0 & 0 & 0 & 1/2 \end{bmatrix}$$

Require that largest eigenvalue respect pw unitarity condition $|a_0| \leq 1$

$$\Longrightarrow \quad M_H \leq \left(\frac{8\pi\sqrt{2}}{3G_F}\right)^{1/2} = 1 \text{ TeV}/c^2$$

condition for perturbative unitarity

[1] Convenient to calculate using *Goldstone-boson equivalence theorem*, which reduces dynamics of longitudinally polarized gauge bosons to scalar field theory with interaction Lagrangian given by $\mathcal{L}_{\text{int}} = -\lambda v h(2w^+w^- + z^2 + h^2) - (\lambda/4)(2w^+w^- + z^2 + h^2)^2$, with $1/v^2 = G_F\sqrt{2}$ and $\lambda = G_F M_H^2/\sqrt{2}$.

- If the bound is respected
  - weak interactions remain weak at all energies
  - perturbation theory is everywhere reliable

- If the bound is violated
  - perturbation theory breaks down
  - weak interactions among $W^{\pm}$, Z, H become strong on 1-TeV scale

  $\Rightarrow$ features of *strong* interactions at GeV energies will characterize *electroweak* gauge boson interactions at TeV energies

  New phenomena are to be found in the EW interactions at energies not much larger than 1 TeV

Threshold behavior of the pw amplitudes a_{IJ} follows from chiral symmetry

$$\begin{array}{lll} a_{00} \approx & G_F s/8\pi\sqrt{2} & \text{attractive} \\ a_{11} \approx & G_F s/48\pi\sqrt{2} & \text{attractive} \\ a_{20} \approx & -G_F s/16\pi\sqrt{2} & \text{repulsive} \end{array}$$

Lee, Quigg, Thacker, *Phys. Rev. D* **16,** 1519 (1977)

What the LHC *is not really* for . . .

- Find the Higgs boson,
  the Holy Grail of particle physics,
  the source of all mass in the Universe.
- Celebrate.
- Then particle physics will be over.

We are not ticking off items on a shopping list . . .

We are exploring a vast new terrain
. . . and reaching the Fermi scale

The Origins of Mass

(masses of nuclei "understood")

$p, [\pi], \rho$ understood: QCD

confinement energy is the source

"Mass without mass" Wilczek, *Phys. Today* (November 1999)

We understand the visible mass of the Universe
. . . without the Higgs mechanism

W, Z electroweak symmetry breaking

$$M_W^2 = \tfrac{1}{2}g^2v^2 = \pi\alpha/G_F\sqrt{2}\sin^2\theta_W$$
$$M_Z^2 = M_W^2/\cos^2\theta_W$$

$q, \ell^\mp$ EWSB + Yukawa couplings

ν_ℓ EWSB + Yukawa couplings; new physics?

All fermion masses $\Leftrightarrow$ physics beyond standard model

H ?? fifth force ??

Challenge: Understanding the Everyday

- Why are there atoms?
- Why chemistry?
- Why stable structures?
- What makes life possible?

What would the world be like, without a (Higgs) mechanism to hide electroweak symmetry and give masses to the quarks and leptons?

Searching for the mechanism of electroweak symmetry breaking, we seek to understand

why the world is the way it is.

This is one of the deepest questions humans have ever pursued, and

it is coming within the reach of particle physics.

Our picture of matter

Pointlike constituents ($r < 10^{-18}$ m)

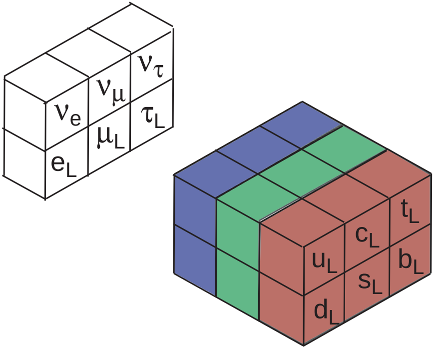
$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} c \\ s \end{pmatrix}_L \quad \begin{pmatrix} t \\ b \end{pmatrix}_L$$

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L$$

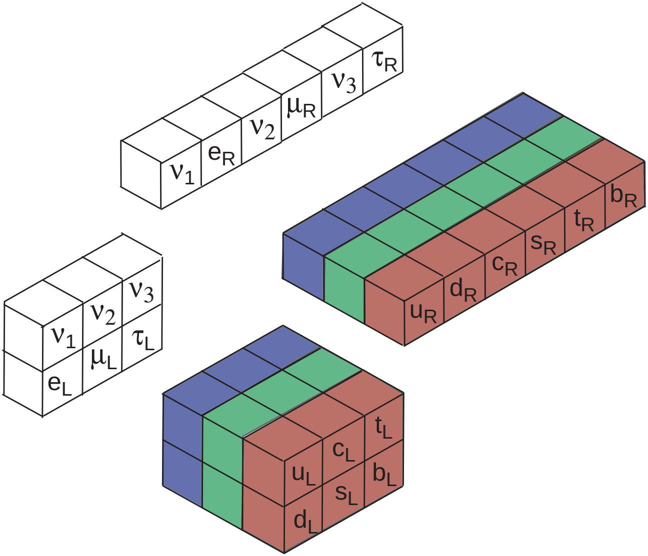
Few fundamental forces, derived from gauge symmetries

$$\mathrm{SU(3)_c} \otimes \mathrm{SU(2)_L} \otimes \mathrm{U(1)}_Y$$

Electroweak symmetry breaking: Higgs mechanism?



ν$_\tau$
ν$_\mu$
ν$_e$
τ$_L$
μ$_L$
e$_L$
t$_L$
c$_L$
u$_L$
b$_L$
s$_L$
d$_L$


ν_1
e_R
ν_2
μ_R
ν_3
τ_R
ν_3
ν_2
ν_1
τ_L
μ_L
e_L
b_R
t_R
s_R
c_R
d_R
u_R
t_L
c_L
u_L
b_L
s_L
d_L

Formulate electroweak theory

Three crucial clues from experiment:

- Left-handed weak-isospin doublets,

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$$

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L \quad \begin{pmatrix} c \\ s' \end{pmatrix}_L \quad \begin{pmatrix} t \\ b' \end{pmatrix}_L \;;$$

- Universal strength of the (charged-current) weak interactions;
- Idealization that neutrinos are massless.

$\boxed{\text{First two clues suggest } SU(2)_L \text{ gauge symmetry}}$

Parity violation in weak decays

1956 Wu *et al.*: correlation between spin vector $\vec{J}$ of polarized ^{60}Co and direction $\hat{p}_e$ of outgoing β particle

Parity leaves spin (axial vector) unchanged $\boxed{\mathcal{P} : \vec{J} \to \vec{J}}$

Parity reverses electron direction $\boxed{\mathcal{P} : \hat{p}_e \to -\hat{p}_e}$

Correlation $\vec{J} \cdot \hat{p}_e$ is *parity violating*

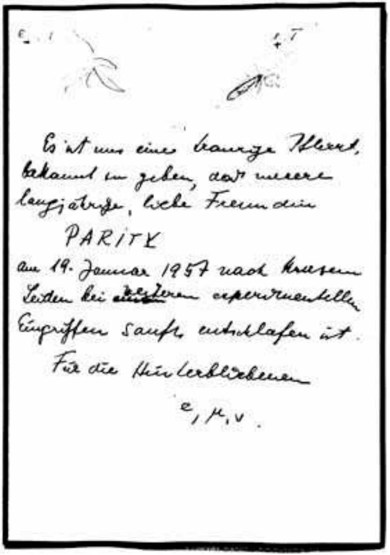
Late 1950s: (charged-current) weak interactions are left-handed

Parity links left-handed, right-handed ν,

$$\nu_L \xrightarrow{\Leftarrow} \mathcal{P} \xleftarrow{\Leftarrow} \cancel{\nu}_R$$

$\Rightarrow$ build a manifestly parity-violating theory with only ν_L.

Pauli's Reaction to the Downfall of Parity



Es ist uns eine traurige Pflicht,
bekannt zu geben, daß unsere
langjährige, liebe Freundin
PARITY
am 19. Januar 1957 nach kurzem
Leiden bei weiteren experimentellen
Eingriffen sanft entschlafen ist.
Für die Hinterbliebenen
e, μ, ν.

Pauli's Reaction to the Downfall of Parity

Es ist uns eine traurige Pflicht, bekannt zu geben, daß unsere langjährige ewige Freundin

PARITY

den 19. Januar 1957 nach kurzen Leiden bei weiteren experimentellen Eingriffen sanfte entschlafen ist.

Für die hinterbliebenen

$e \quad \mu \quad \nu$

It is our sad duty to announce that our loyal friend of many years

PARITY

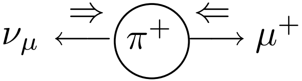
went peacefully to her eternal rest on the nineteenth of January 1957, after a short period of suffering in the face of further experimental interventions.

For those who survive her,

$e \quad \mu \quad \nu$

How do we know ν is left-handed?

▷ $\boxed{\nu_\mu}$ Measure μ^+ helicity in (spin-zero) $\pi^+ \to \mu^+\nu_\mu$



$\boxed{h(\nu_\mu) = h(\mu^+)}$ Bardon, PRL **7,** 23 (1961); Possoz, PL **70B,** 265 (1977)

μ^+ forced to have "wrong" helicity

...inhibits decay, and inhibits $\pi^+ \to e^+\nu_e$ more

$$\Gamma(\pi^+ \to e^+\nu_e)/\Gamma(\pi^+ \to \mu^+\nu_\mu) = 1.23 \times 10^{-4}$$

▷ Longitudinal pol. of recoil nucleus in $\mu^- {}^{12}\mathrm{C}(J=0) \to {}^{12}\mathrm{B}(J=1)\nu_\mu$

$\boxed{\text{Infer } h(\nu_\mu) \text{ by angular momentum conservation}}$

Roesch, *Am. J. Phys.* **50**, 931 (1981)

▷ $\boxed{\nu_e}$ Measure longitudinal polarization of recoil nucleus in

$$e^- \, {}^{152}\mathrm{Eu^m}(J=0) \;\; \rightarrow \;\; {}^{152}\mathrm{Sm}^*(J=1)\, \nu_e$$
$$\hookrightarrow \gamma \, {}^{152}\mathrm{Sm}$$

$\boxed{\text{Infer } h(\nu_e) \text{ from } \gamma \text{ polarization}}$

Goldhaber, *Phys. Rev.* **109**, 1015 (1958)

▷ $\boxed{\nu_\tau}$ Variety of determinations in $\tau \rightarrow \pi\nu_\tau$, $\tau \rightarrow \rho\nu_\tau$, etc.

e.g., Abe, *et al.* (SLD), *Phys. Rev. Lett.* **78**, 4691 (1997)

Charge conjugation is also violated . . .

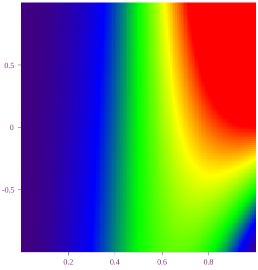
$$\nu_L \xrightarrow{\Leftarrow} \mathcal{C} \xrightarrow{\Leftarrow} \cancel{\bar{\nu}_L}$$

$\mu^{\pm}$ decay: angular distributions of $e^{\pm}$ reversed

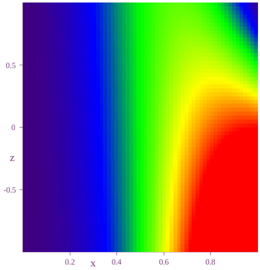
$$\frac{dN(\mu^{\pm} \to e^{\pm} + \ldots)}{dxdz} = x^2(3-2x)\left[1 \pm z\frac{(2x-1)}{(3-2x)}\right]$$

$$x \equiv p_e/p_e^{\max},\; z \equiv \hat{s}_\mu \cdot \hat{p}_e$$

e^+ follows μ^+ spin



e^- avoids μ^- spin



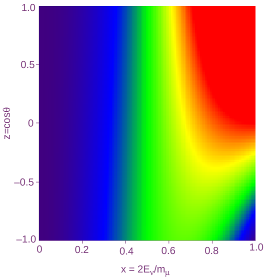
Consequences for neutrino factory

$$\mu^+ \to e^+ \bar{\nu}_\mu \nu_e$$

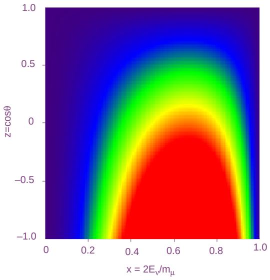
$$\frac{d^2 N_{\bar{\nu}_\mu}}{dxdz} = x^2[(3-2x)-(1-2x)z]\,, \quad x \equiv p_\nu / p_\nu^{\max},\ z \equiv \hat{p}_\nu \cdot \hat{s}_\mu$$

$$\mu^+ \to e^+ \bar{\nu}_\mu \nu_e$$

$$\frac{d^2 N_{\nu_e}}{dxdz} = 6x^2[(1-x)(1-z)]$$



$\boxed{\bar{\nu}_\mu}$



$\boxed{\nu_e}$

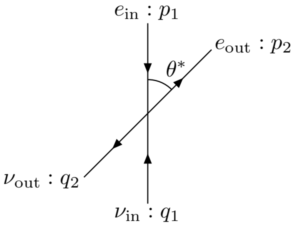
Effective Lagrangian . . .

Late 1950s: current-current interaction

$$\mathcal{L}_{V-A} = \frac{-G_F}{\sqrt{2}} \bar{\nu}\gamma_\mu(1-\gamma_5)e \; \bar{e}\gamma^\mu(1-\gamma_5)\nu + \mathrm{h.c.}$$

$$G_F = 1.16632 \times 10^{-5}\ \mathrm{GeV}^{-2}$$

Compute $\bar{\nu}e$ scattering amplitude:



$$\begin{aligned}\mathcal{M} \;\; &= \;\; -\frac{iG_F}{\sqrt{2}} \bar{v}(\nu, q_1)\gamma_\mu(1-\gamma_5)u(e, p_1) \\ &\quad \cdot \bar{u}(e, p_2)\gamma^\mu(1-\gamma_5)v(\nu, q_2)\end{aligned}$$

$\boxed{\bar{\nu} e \to \bar{\nu} e}$

$$\frac{d\sigma_{V-A}(\bar{\nu} e \to \bar{\nu} e)}{d\Omega_{\text{cm}}} = \frac{\overline{|\mathcal{M}|^2}}{64\pi^2 s} = \frac{G_F^2 \cdot 2mE_\nu (1-z)^2}{16\pi^2} \quad z = \cos\theta^*$$

$$\begin{aligned} \sigma_{V-A}(\bar{\nu} e \to \bar{\nu} e) &= \frac{G_F^2 \cdot 2mE_\nu}{3\pi} \\ &\approx 0.574 \times 10^{-41}\ \text{cm}^2 \left(\frac{E_\nu}{1\ \text{GeV}}\right) \end{aligned}$$

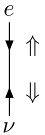
Small! $\approx 10^{-14}\ \sigma(pp)$ at 100 GeV

$\boxed{\nu e \to \nu e}$

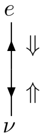
$$\frac{d\sigma_{V-A}(\nu e \to \nu e)}{d\Omega_{\text{cm}}} = \frac{G_F^2 \cdot 2mE_\nu}{4\pi^2}$$

$$\begin{aligned} \sigma_{V-A}(\nu e \to \nu e) &= \frac{G_F^2 \cdot 2mE_\nu}{\pi} \\ &\approx 1.72 \times 10^{-41}\ \text{cm}^2 \left(\frac{E_\nu}{1\ \text{GeV}}\right) \end{aligned}$$

Why 3× difference?

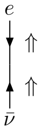
incoming 

$J_z = 0$

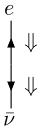
outgoing, $z = +1$ 

$J_z = 0$

allowed at all angles

incoming 

$J_z = +1$

outgoing, $z = +1$ 

$J_z = -1$

forbidden (angular momentum) at $z = +1$

1962: Lederman, Schwartz, Steinberger $\nu_\mu \neq \nu_e$

- ▷ Make HE $\pi \to \mu\nu$ beam
- ▷ Observe $\nu N \to \mu +$ anything
- ▷ Don't observe $\nu N \to e +$ anything

Danby, *et al.*, *Phys. Rev. Lett.* **9**, 36 (1962)

Suggests family structure

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L$$

$\approx$ no interactions known to cross boundaries

Generalize effective (current-current) Lagrangian:

$$\mathcal{L}_{V-A}^{(e\mu)} = \frac{-G_F}{\sqrt{2}} \bar{\nu}_\mu \gamma_\mu (1-\gamma_5)\mu \; \bar{e}\gamma^\mu(1-\gamma_5)\nu_e + \mathrm{h.c.}\;,$$

Compute muon decay rate

$$\Gamma(\mu \to e\bar{\nu}_e\nu_\mu) = \frac{G_F^2 m_\mu^5}{192\pi^3}$$

accounts for the 2.2-μs muon lifetime

Cross section for inverse muon decay

$$\sigma(\nu_\mu e \to \mu \nu_e) = \sigma_{V-A}(\nu_e e \to \nu_e e)\left[1 - (m_\mu^2 - m_e^2)/2m_e E_\nu\right]^2$$

agrees with CHARM II, CCFR data ($E_\nu \lesssim 600$ GeV)

$\boxed{\text{PW unitarity: } |\mathcal{M}_J| < 1}$

$$V - A \text{ theory:} \quad \mathcal{M}_0 = \frac{G_F \cdot 2m_e E_\nu}{\pi\sqrt{2}}\left[1 - \frac{(m_\mu^2 - m_e^2)}{2m_e E_\nu}\right]$$

satisfies pw unitarity for

$$E_\nu < \pi / G_F m_e \sqrt{2} \approx 3.7 \times 10^8 \text{ GeV}$$

$\boxed{\Rightarrow V - A \text{ theory cannot be complete}}$

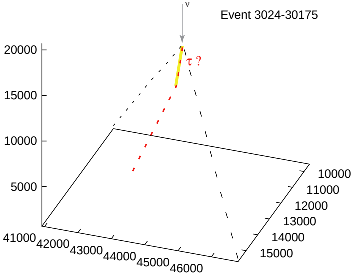
Physics must change below $\sqrt{s} \approx 600$ GeV

2000: DONuT Three-Neutrino Experiment

- Prompt (beam-dump) ν_τ beam produced in

$$D_s^+ \rightarrow \tau^+\nu_\tau \qquad \tau^+ \hookrightarrow X^+\bar{\nu}_\tau$$

- Observe $\nu_\tau N \rightarrow \tau +$ anything in emulsion; τ lifetime is 0.3 ps



Candidate event in ECC1. The three tracks with full emulsion data are shown. The red track shows a 100 mrad kink 4.5mm from the interaction vertex. The scale units are microns.

Kodama, *et al.*, *Phys. Lett.* **B504**, 218 (2001)

Leptons are seen as free particles

Table: Some properties of the leptons.

Lepton	Mass	Lifetime
ν_e	< 2 eV	
e^-	$0.510\,998\,918(44)$ MeV	$> 4.6 \times 10^{26}$ y (90% CL)
ν_μ	< 0.19 MeV (90% CL)	
μ^-	$105.658\,369\,2(94)$ MeV	$2.197\,03(4) \times 10^{-6}$ s
ν_τ	< 18.2 MeV (95% CL)	
τ^-	1776.90 ± 0.20 MeV	$290.6 \pm 1.0 \times 10^{-15}$ s

All spin-$\frac{1}{2}$, pointlike ($\lesssim$ few $\times\, 10^{-17}$ cm)

kinematically determined ν masses consistent with 0
(ν oscillations $\Rightarrow$ nonzero, unequal masses)

Universal weak couplings: *Rough and ready test*

Fermi constant from muon decay

$$G_\mu = \left[\frac{192\pi^3\hbar}{\tau_\mu m_\mu^5}\right]^{\frac{1}{2}} = 1.1638 \times 10^{-5}\ \mathrm{GeV}^{-2}$$

Meticulous analysis yields $G_\mu = 1.16637(1) \times 10^{-5}\ \mathrm{GeV}^{-2}$

Fermi constant from tau decay

$$G_\tau = \left[\frac{\Gamma(\tau \to e\bar{\nu}_e\nu_\tau)}{\Gamma(\tau \to \mathrm{all})}\frac{192\pi^3\hbar}{\tau_\tau m_\tau^5}\right]^{\frac{1}{2}} = 1.1642 \times 10^{-5}\ \mathrm{GeV}^{-2}$$

Excellent agreement with $G_\beta = 1.16639(2) \times 10^{-5}\ \mathrm{GeV}^{-2}$

Charged currents acting in leptonic and semileptonic interactions are of universal strength; $\Rightarrow$ *universality of current-current form, or whatever lies behind it*

Nonleptonic enhancement

Certain NL transitions are more rapid than universality suggests

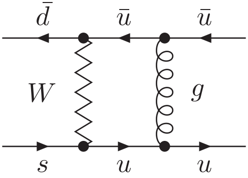
$$\underbrace{\Gamma(K_S \to \pi^+\pi^-)}_{I=0,2} \approx 450 \times \underbrace{\Gamma(K^+ \to \pi^+\pi^0)}_{I=2}$$

$$\boxed{A_0 \approx 22 \times A_2}$$

$|\Delta I| = \frac{1}{2}$ rule; "octet dominance" (over **27**)

Origin of this phenomenological rule is only partly understood.

Short-distance *(perturbative)* QCD corrections arise from



... explain $\approx \sqrt{\text{enhancement}}$

A theory of leptons

$$\mathsf{L} = \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \qquad \mathsf{R} \equiv e_R$$

weak hypercharges $Y_L = -1$, $Y_R = -2$

Gell-Mann–Nishijima connection, $Q = I_3 + \frac{1}{2}Y$

$\mathrm{SU(2)_L} \otimes \mathrm{U(1)}_Y$ gauge group $\Rightarrow$ gauge fields:

- weak isovector $\vec{b}_\mu$, coupling g $\quad \boxed{b_\mu^\ell = b_\mu^\ell - \varepsilon_{jk\ell}\alpha^j b_\mu^k - (1/g)\partial_\mu \alpha^\ell}$
- weak isoscalar $\mathcal{A}_\mu$, coupling $g'/2$ $\quad \boxed{\mathcal{A}_\mu \rightarrow \mathcal{A}_\mu - \partial_\mu \alpha}$

Field-strength tensors

$$F_{\mu\nu}^\ell = \partial_\nu b_\mu^\ell - \partial_\mu b_\nu^\ell + g\varepsilon_{jk\ell} b_\mu^j b_\nu^k \;, SU(2)_L$$

$$f_{\mu\nu} = \partial_\nu \mathcal{A}_\mu - \partial_\mu \mathcal{A}_\nu \;, U(1)_Y$$

Interaction Lagrangian

$$\boxed{\mathcal{L} = \mathcal{L}_{\text{gauge}} + \mathcal{L}_{\text{leptons}}}$$

$$\mathcal{L}_{\text{gauge}} = -\tfrac{1}{4}F^{\ell}_{\mu\nu}F^{\ell\mu\nu} - \tfrac{1}{4}f_{\mu\nu}f^{\mu\nu},$$

$$\begin{aligned}\mathcal{L}_{\text{leptons}} &= \overline{\mathsf{R}}\, i\gamma^{\mu}\left(\partial_{\mu} + i\frac{g'}{2}\mathcal{A}_{\mu}Y\right)\mathsf{R} \\ &+ \overline{\mathsf{L}}\, i\gamma^{\mu}\left(\partial_{\mu} + i\frac{g'}{2}\mathcal{A}_{\mu}Y + i\frac{g}{2}\vec{\tau}\cdot\vec{b}_{\mu}\right)\mathsf{L}.\end{aligned}$$

Mass term $\mathcal{L}_e = -m_e(\bar{e}_{\mathsf{R}}e_{\mathsf{L}} + \bar{e}_{\mathsf{L}}e_{\mathsf{R}}) = -m_e\bar{e}e$ violates local gauge inv.

Theory: 4 massless gauge bosons ($\mathcal{A}_{\mu} \quad b^1_{\mu} \quad b^2_{\mu} \quad b^3_{\mu}$); Nature: 1 ($\gamma$)

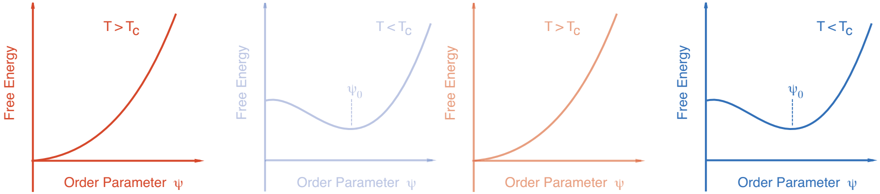
Massive Photon? *Hiding Symmetry*

Recall $\boxed{2}$ miracles of superconductivity:

- No resistance . . . . . . Meissner effect (exclusion of **B**)

Ginzburg–Landau Phenomenology (not a theory from first principles)

normal, resistive charge carriers . . . . . . + superconducting charge carriers



$$\mathbf{B} = 0: \qquad G_{\text{super}}(0) = G_{\text{normal}}(0) + \alpha\,|\psi|^2 + \beta\,|\psi|^4$$

$$T > T_c: \quad \alpha > 0 \quad \langle|\psi|^2\rangle_0 = 0$$

$$T < T_c: \quad \alpha < 0 \quad \langle|\psi|^2\rangle_0 \neq 0$$

In a nonzero magnetic field ...

$$G_{\text{super}}(\mathbf{B}) = G_{\text{super}}(0) + \frac{\mathbf{B}^2}{8\pi} + \frac{1}{2m^*}\left|-i\hbar\nabla\psi - \frac{e^*}{c}\mathbf{A}\psi\right|^2$$

$$\left.\begin{array}{c} e^* = -2 \\ m^* \end{array}\right\} \text{of superconducting carriers}$$

Weak, slowly varying field: $\psi \approx \psi_0 \neq 0,\ \nabla\psi \approx 0$

Variational analysis $\rightsquigarrow$

$$\boxed{\nabla^2\mathbf{A} - \frac{4\pi e^{*2}}{m^* c^2}\left|\psi_0\right|^2\mathbf{A} = 0}$$

wave equation of a *massive photon*

Photon – *gauge boson* – acquires mass
within superconductor

origin of Meissner effect

Meissner effect levitates Leon Lederman (Snowmass 2001)

Hiding EW Symmetry

Higgs mechanism: relativistic generalization of Ginzburg-Landau superconducting phase transition

- Introduce a complex doublet of scalar fields

$$\phi \equiv \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \quad Y_\phi = +1$$

- Add to $\mathcal{L}$ (gauge-invariant) terms for interaction and propagation of the scalars,

$$\mathcal{L}_{\text{scalar}} = (\mathcal{D}^\mu \phi)^\dagger (\mathcal{D}_\mu \phi) - V(\phi^\dagger \phi),$$

  where $\mathcal{D}_\mu = \partial_\mu + i\frac{g'}{2}\mathcal{A}_\mu Y + i\frac{g}{2}\vec{\tau} \cdot \vec{b}_\mu$ and

$$\boxed{V(\phi^\dagger \phi) = \mu^2(\phi^\dagger \phi) + |\lambda| (\phi^\dagger \phi)^2}$$

- Add a Yukawa interaction $\mathcal{L}_{\text{Yukawa}} = -\zeta_e \left[\overline{\mathsf{R}}(\phi^\dagger \mathsf{L}) + (\overline{\mathsf{L}}\phi)\mathsf{R}\right]$

- Arrange self-interactions so vacuum corresponds to a broken-symmetry solution: $\mu^2 < 0$
  Choose minimum energy (vacuum) state for vacuum expectation value

$$\langle\phi\rangle_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix}, \quad v = \sqrt{-\mu^2/|\lambda|}$$

Hides (breaks) $SU(2)_L$ and $U(1)_Y$

but preserves $U(1)_{\mathrm{em}}$ invariance

Invariance under $\mathcal{G}$ means $e^{i\alpha\mathcal{G}}\langle\phi\rangle_0 = \langle\phi\rangle_0$, so $\mathcal{G}\langle\phi\rangle_0 = 0$

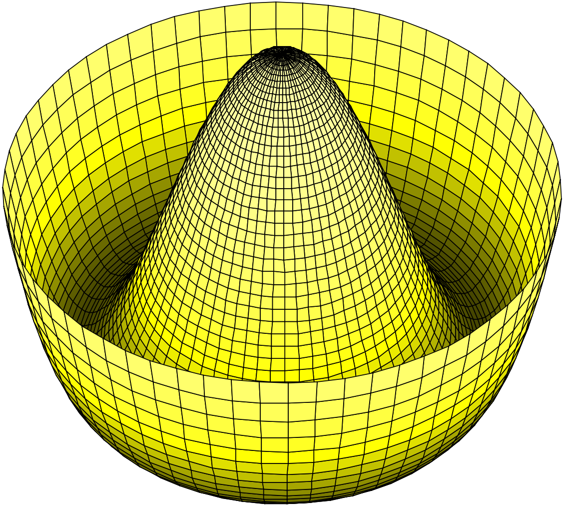
$$\tau_1\langle\phi\rangle_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix} \neq 0 \quad \text{broken!}$$

$$\tau_2\langle\phi\rangle_0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} -iv/\sqrt{2} \\ 0 \end{pmatrix} \neq 0 \quad \text{broken!}$$

$$\tau_3\langle\phi\rangle_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ -v/\sqrt{2} \end{pmatrix} \neq 0 \quad \text{broken!}$$

$$Y\langle\phi\rangle_0 = Y_\phi\langle\phi\rangle_0 = +1\langle\phi\rangle_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \neq 0 \quad \text{broken!}$$

Symmetry of laws $\not\Rightarrow$ symmetry of outcomes



Examine electric charge operator Q on the (neutral) vacuum

$$\begin{aligned}
Q\langle\phi\rangle_0 &= \tfrac{1}{2}(\tau_3 + Y)\langle\phi\rangle_0 \\
&= \tfrac{1}{2}\begin{pmatrix} Y_\phi + 1 & 0 \\ 0 & Y_\phi - 1 \end{pmatrix}\langle\phi\rangle_0 \\
&= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \\
&= \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \textit{unbroken!}
\end{aligned}$$

Four original generators are broken, *electric charge is not*

- $\mathrm{SU(2)_L} \otimes \mathrm{U(1)}_Y \rightarrow\ U(1)_{\mathrm{em}}$ (will verify)
- Expect massless photon
- Expect gauge bosons corresponding to

$$\tau_1,\ \tau_2,\ \tfrac{1}{2}(\tau_3 - Y) \equiv K \quad \text{to acquire masses}$$